Fluid Mechanics for Civil and Mechanical Engineering
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Lecture No 11
Bernoulli Equation

Welcome all of you this mock course on a fluid mechanics. Today will have very interesting topic that is what Bernoulli's equations. As of now you know it, the very initially, I discussed about Newton's laws of viscosities, then we discuss about the Reynold's transport theorem. The Bernoulli's equation is another conclusion by Bernoulli is used in different fluid flow problems. In short, I can tell you that these equations help us to solve the many fluid flow problems by considering energy considerations or a linear momentum equation.

And this is very easy equations in terms to incorporate between the energy losses absorbed from the experimental data to analytical framework. So, that is the reasons this equation mostly used for fluid flow problems, because this equation looks as energy conservations equations. Also, it looks like the linear momentum equations along a streamlines. Let us have a discussion today on these equations, Bernoulli equations. I have prepared this lecture in very interesting way.

Let us look at these lectures in very positive way to understand what is the Bernoulli equations. As I know, at the class 11th or 12th level you know what is Bernoulli equations, but many of the times you do not know when you apply the Bernoulli equations, what are the assumptions behind that? Is it energy conservation equations or linear momentum equations. That questions also posses us whenever we this energy conservation equation.

Because always we need to solve the pressure field and the velocity field, that is what I discussed earlier. Whenever we talk about fluid mechanics, we talk about two fields are important for us for incompressible flow, that is our pressure field and the velocity field. But how we can simplify these two fields using these Bernoulli equations, that is the strength of the Bernoulli equations and that is the reason this equations are extensively used to solve the fluid flow problems.

Not only that, as you know, we have now the fluid mechanics solvers like computational fluid dynamics, lot of things have improved, improved mechanics in terms of computational fluid dynamics, in terms of experiments. But whoever the fluid mechanics specialist, he first look is two conservation principles, one is mass conservation, second he uses the Bernoulli equations to verify the results, either from experimental or from the results obtained from computational fluid dynamics.

So, that is the reasons, even if we have today, very advanced tools with us, but these equations really have a lot of applications for us to check their results, are they correct or not. With this brief introductions, let I go to the today contents of the lectures. We will start with applications because, as I know, you know, what is Bernoulli equations. What is the terms in the Bernoulli equations.

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I will start with the applications, then I will go for Bernoulli experiment. That is what is we did in fluid mechanics lab in IIT Guwahati, then I will come into theoretical derivations of Bernoulli equations, that is the part what we will cover it and we will have theoretical derivations of Bernoulli equations and then what will follow it, again I will come back to explaining this Bernoulli equations using virtual fluid ball concept. And then you can easily visualize the fluid problems.

And we will solve some of the simple example problems using Bernoulli equations. At the last, I will talk about the sense of balance what we have in the human body, okay and we will have a summary. So, I will start from applications, then go for experiment, then the derivations of Bernoulli equations. And then I will talk about the virtual fluid ball concept, what I introduced for you and some two examples will just demonstrate it, how we can use the Bernoulli equations with mass conservations equations.

Then I will talk about sense of the balance and the summary. Let us look at very interestingly, okay.

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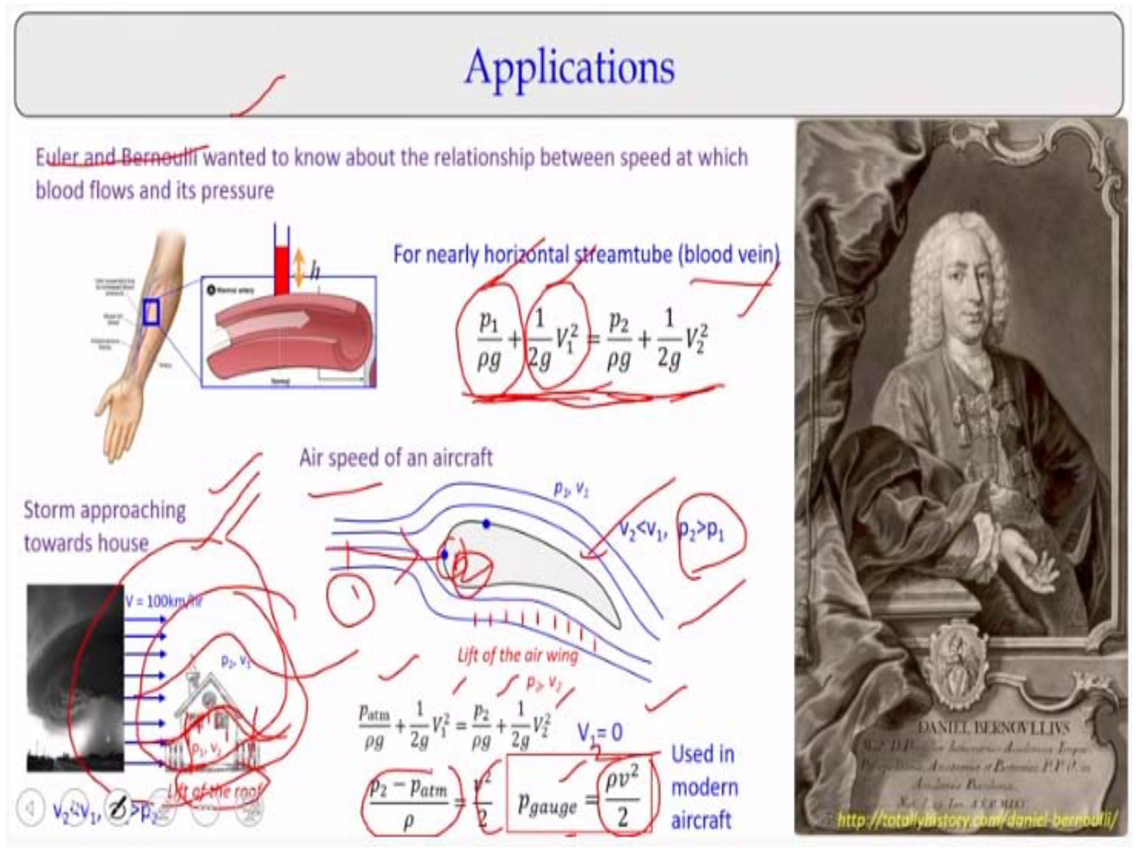

I can say it the contributions of Daniel Bernoulli, we cannot deny it, where he revised these simple formulations of Bernoulli equations help us to modernize all these chemical industry by designing the pipe networks. He also helped to visualize the how we can get it the lifting force from the any air foils as a part of the air wing. So his contributions in terms of chemical industry or the green industrial revolutions, we can we can really acknowledge his contributions in terms of these, not only these equations, also, other components.

Those who are interested, I can just encourage you please visit Wikipedia of Daniel Bernoulli and his contributions. Let us start with the Wikipedia's information what is available is that, that initially the Euler and Bernoulli, they wanted to know between the relationship between the speed at which blood clots and its pressure. So, if you look it, the Euler and Bernoulli both they tried to look at what could be the relationship between the speed at which blood flows.

That means velocity and its pressure. That is what is very important when you want to find out the disease, okay, the symptoms of a disease we look it from the blood pressures or we look at the blood flow. Very simple way you know it, at the very beginning of your 12th class that the equations has three components, total energy has three components. One is the flow energy component, another is the kinetic energy component. And if you consider that your blood vessels are in a near horizontal conditions.

Then, we have **(()) (08:26)** sections if you have a flow energy per weight by kinetic energy per weight is equal to the flow energy and the kinetic energy per weight. So that is the reason, if you look

For nearly horizontal streamtube (blood vein)

$$\frac{p_1}{\rho g} + \frac{1}{2g}V_1^2 = \frac{p_2}{\rho g} + \frac{1}{2g}V_2^2$$

We can look it, there is a relationship between the pressure and the velocity. So, wherever the pressure increases, the definite the velocity has to decrease or the velocity increases, the pressure decreases.

So, these are very common flow problems observed long ago that wherever in a fluid flow problems when a fluid is going on, if there is increase of the pressure, definitely there will be a decrease of the velocity to make it the energy conservations along this fluid flow problems. So, if you look it with that concept, that the speed at which blood flows and its pressure, they have a inverse relationship. If pressure increases, the velocity decreases, if velocity increases then the pressure decreases.

So, those relationships, you can look it from this relationship, which is observed many of the times, okay. Most often I can go back to the cyclonic disasters, as you know it, what it happens is, most of the time is blown off the rooftops. Why it happens? If look at these tops, okay? Why does this happen? Whenever you have the, let us have the wind having a velocity of 100 kilometer per hour, as soon as it comes, you will have a one streamline will go like this.

Another flow will anchor it and go inside it like that. So, you have a two streamlines of this, okay. As you can understand it, the velocity of the point which is a p1 V1 and p2 V2, the velocity decreases, just below of this point, the pressure is increases as compared to the p2. So, net pressure gradient multiplied to the area will give us a lift force and this lift force work to blown off this rooftop. So, if you look at this process, that how a rooftop is blown off during the cyclonic storms, you can understand it.

So, most of the times what people they do it, they close the window and doors such a way that there will be no the velocity reduction or the high pressure zone formations inside the house. If that conditions prevails, then there will be no blown off the rooftops. So, if you look at this way, whenever we design a house near a coastal belt, then we should take care of the wind directions, we should take care of to know it, what is the wind velocity is coming at.

Similar way, if you look it, we take advantage of the change of the pressure and the velocity gradient. As you know it that as the pressure is going like this, over an aerofoil, which is representing a part of section of the wing of an aircraft. So, what it indicates that, there is the reductions of the velocity in the lower part. Because of that, there will be the pressure increase. As compared to that, your p_1 V_1 will be the lesser. So V2 will be lesser than V1, the pressures will be greater than p1.

$v_2 < v_1$, $p_2 > p_1$

So, because of that, there will be a net force is going to work it upward direction, that is what uplift force what is acts is. So one way we use the uplift force for lifting an aircraft, another way it is a disaster for us whenever you have the velocity reductions and increase of the pressure inside a house and you have a cyclonic storms is going on. So there will be lift force is acting on this rooftop and that is what will be going to blown off these things.

So, if you look at the three mechanisms from a blot bend to the lifting of roof or lifting a wing of an aircraft, all are linked and we solved with a very simple concept of Bernoulli equations. Now, let us look at that I want to have to compute, what could be the air speed okay. So that means, I can draw a streamlines which hitting this, okay. So this is what the streamline hitting over this and if I apply this streamlines, because it is nearly horizontal.

I can apply this again, the Bernoulli equations at two point; point one and point two. And if I have a point one and the point two, where I can measure the pressures, which is the point which is called stagnation point, the point where the velocity becomes zero.

$$\frac{p_{\text{atm}}}{\rho g} + \frac{1}{2g}V_1^2 = \frac{p_2}{\rho g} + \frac{1}{2g}V_2^2$$

When the velocity becomes zero, in that case, that is what the case which is $V_2 = 0$, the velocity is equal to zero. In that case, what we will have, we will find out this gauge pressure difference,

$$\frac{p_2 - p_{atm}}{\rho} = \frac{v^2}{2}$$

$$p_{gauge} = \frac{\rho v^2}{2}$$

So, if this is a simple relationship, if you measure the gauge pressure, you can compute the velocity or the air speed. So, this simple concept of having a pressure sensor on the air wing to compute air speed still also used in modern aircraft. So, if you look at the applications are Bernoulli equations, it is really good equations we can utilized it properly that then we can derive how the presser and velocity variations. And so once you know the pressure and velocity variations, integrating over the area or along the streamlines.

We can find out what could be the lift force, drag force, all the components we can compute it. But it has a lot of assumptions. So, let us start looking it for an experiment.

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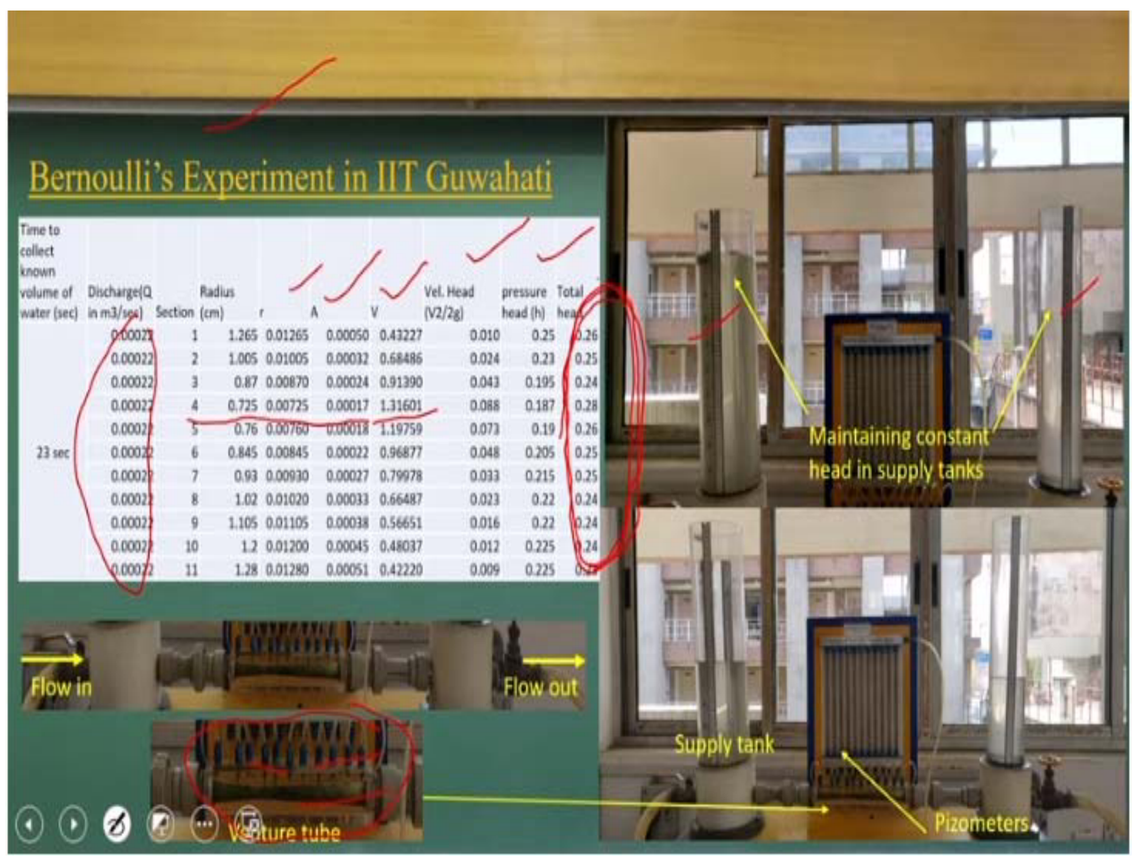

This is what the verification of Bernoulli equations in fluid mechanics lab in IIT, Guwahati. If you look at these ones, it has two tubes to maintaining the contents in supply heads. There is two tanks are there, which is maintaining the supply heads and here, you have a venture meters. We can very clearly look it, the cross sections area is decreasing it after certain times, again the cross section area is increasing.

So, there is a decreasing of the cross section area, then increasing. So, you have a converging zone,, then it follows diversion gauge zone. So, if you a conduct series of pressure meters which measure the pressures or the velocity head. If you look at that, this venture meters have a converging zones and the diverging zones. And these are a series of the pressure meters are there, we can measure the pressure head.

And as we know, for a particular discharge, constant discharge, we can have a 11 pressure meters locations, where the radius are different, area of the flow is different, and the velocity will be different. That is what, so discharge is same since the area of the flow is varies. So you have the increase of the velocity and the decrease of velocity. So, you can look it, we have with the high velocity zone, where we have less cross-section area.

This very simple things, the discharge is equal to area into velocity. If area decreases velocity increases, that is what we got it. And we measure the velocity head, compute the velocity head, we have with a pressure head. Equating these, if you look it, more or less the total head is the same, that means we have verified it that the total head along a streamline when going through a venture meters, it have the same value. A slight bit difference will be there, there is energy loss phenomenon is happening.

But we can consider that this is more or less constant value. The total head, the pressure head and the velocity head is constant because they are constant, the nearly horizontal flow systems. So, this way, we can verify the Bernoulli equations sequence with these apparatus. Now, let us come to the derivations of Bernoulli equations, which was derived by the Euler long back in 1755.

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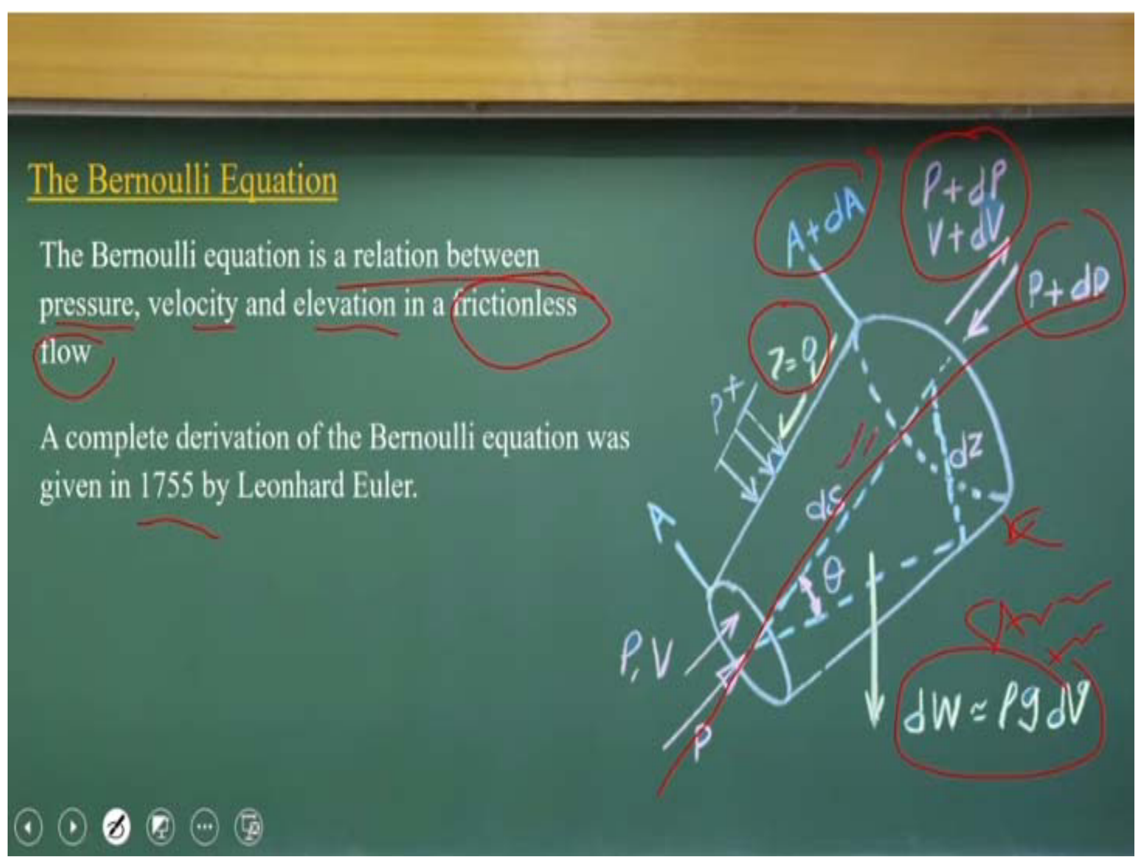

The complete derivations of Bernoulli equation, which is very complicated. But very simple way, if I am to derive the Bernoulli equation, which gives us a relationship between the pressure, velocity, elevations in a frictionless flow which is very idealized conditions. That means there is no shear stress component, okay. There is no shear stress component. We are assuming it that I have a simple, a streamline going on and along this streamline is flow is happening and these control volumes representing me a stream tube.

So, if this is my control volume, I am trying to apply basic mass conservation equations and linear momentum equations. Here, what are the forces acting it, one is wet and there is a pressure difference between these two places, the p is varies from this point to this. Here it is p, here p + dp and there is a variations of velocity p to V + dV and area is also increases along the streamlines A + dA. So, we have a proximal relationship of pressure, the velocity, density and area which are changing along the streamlines.

And the streamlines are noted here as S. The directional components what we have given as S component.

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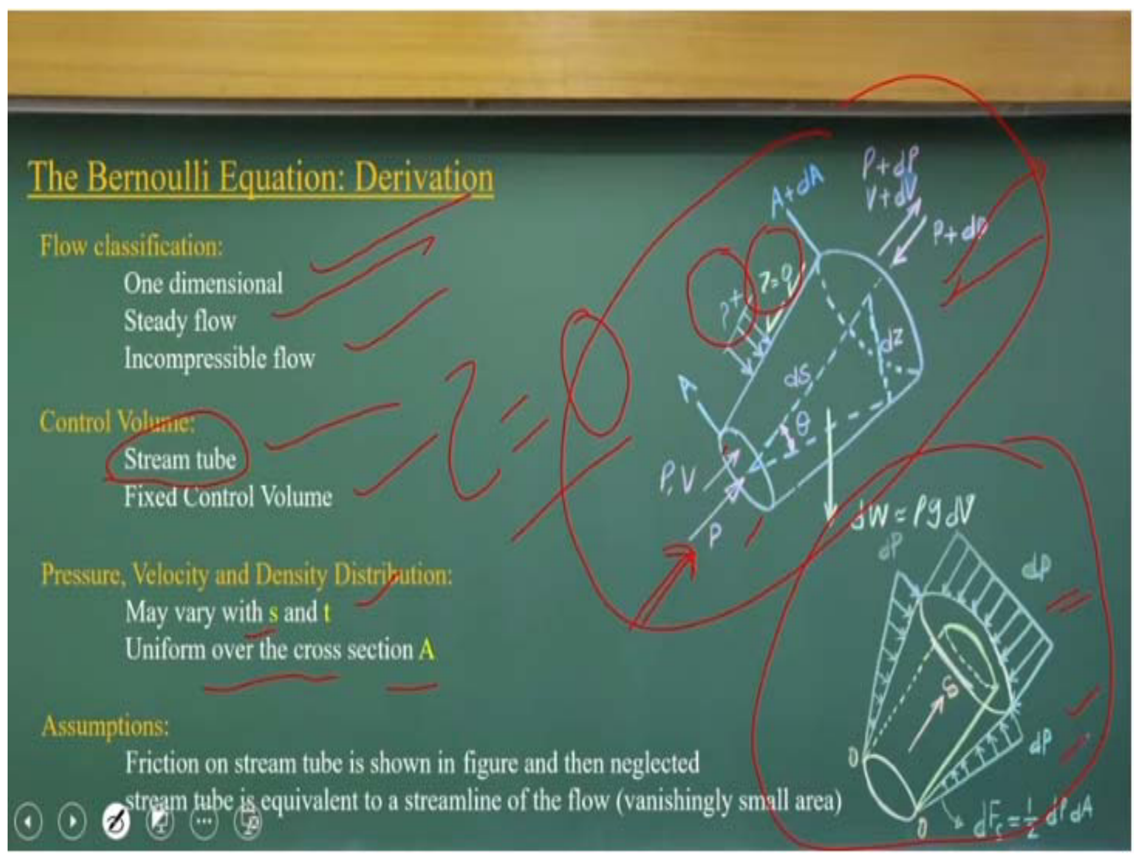

Now, let us have a the flow classifications or the simplifications that in this case we have considered one dimensional steady incompressible or compressible that is what we will discuss just a bit here. We have a stream tube, fixed control volume, okay. So, basically, as I told earlier, we have considering a stream tube concept, okay. That means for a steady flow, we have a series of this streamlines are there, defining their space along the stream tube.

The advantage of the stream tube is that there is no flow across this stream tube, because that is what the definitions of the streamlines, there will be no flow component to normal to this part, because all the flow components will be tangential to the spot. So, there is no velocity gradient perpendicular to that. Because of that, we do not have any flow pass through perpendicular to the surface. The flow comes from this direction and goes from this.

The mass inflow comes from this directions and goes from this direction. Similar way, the momentum flux coming into this surface, going out from this surface. So, now the problem are easy as we have considered the steam tube as a control volume. Please try to look at, what we have consider is that the stream tube is a control volume. Because of that, over the surface we do not have a mass influx coming into this control volume as well as there is no momentum flux coming into this control volume through this surface.

Only it is coming from inflow and the outflow surface, the problems becomes now a bit easier as compared to taking an arbitrary control volumes. So, the control volume what you have consider it, please remember is a stream tube. So, the pressure, velocity and density, all they are varying with respect to S, S is the directions along the streamline and the T is a time component. And we can consider the velocity and the pressure at inflow and the outflow levels are more less uniform.

We are not considering the velocity variations at the cross section levels. Along the streamline, it varies, but it is uniform when you consider equating the pressure. Now if you look it, since there is no friction is there, okay. So, we can easily consider is that shear stress is equal to zero. So there is shear force component is coming over the surface. So, again we have a quite simplification that. But if we look at, if I narrow down, the reduce the size of the stream tube smaller at a particular level.

So, I can see that is equivalent to a streamloc. Now let us look at the pressure distribution. This is what how the pressure various are there. See this pressure is here, so different pressure if I look it or the pressure difference between inflow and outflow. Here this pressure is dp, but on the peripheral of these control volumes where we have no momentum flux, no mass flux, the shear stress acting over the surface is zero, but there will be a pressure which we can consider a linearly increasing it from zero from here to dp.

Because we want to look at the pressure difference diagrams to find out what is the pressure force course acting on this control volume. Now let us have the basic definitions.

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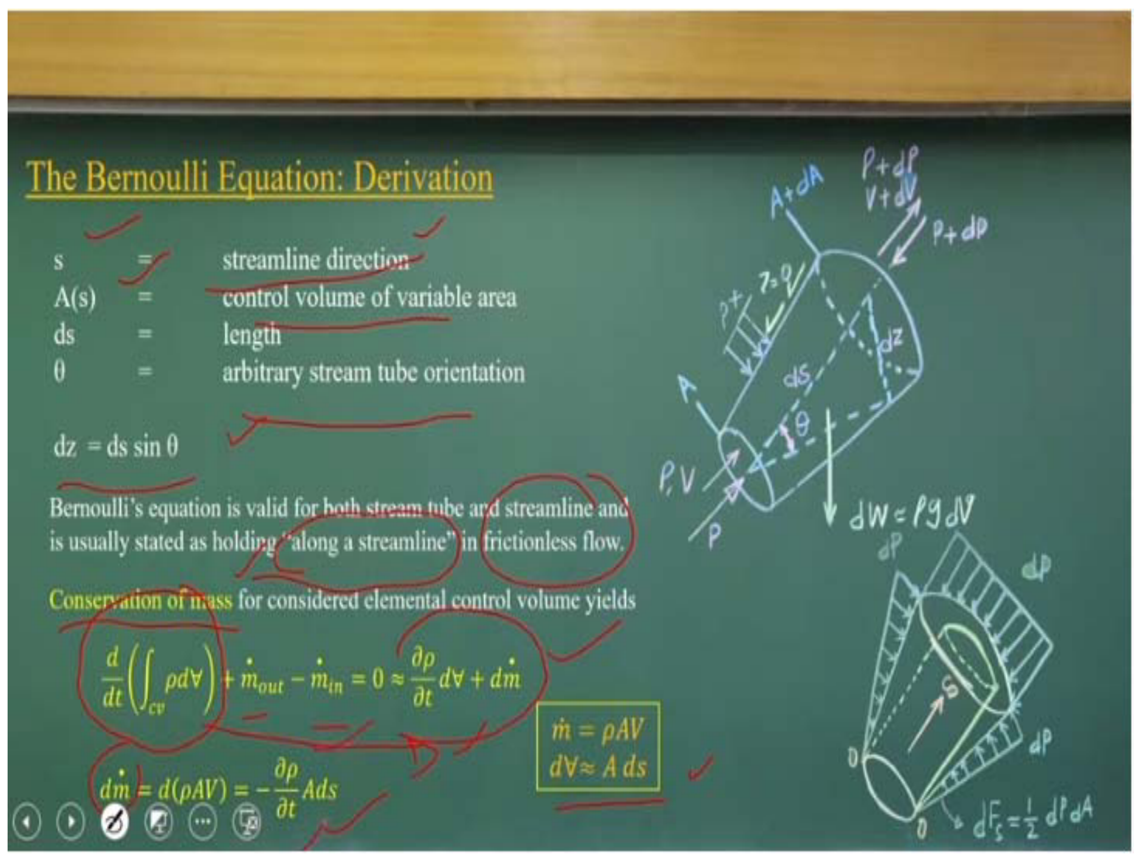

As I say that, s stands for the streamline directions, As stands for controllable variable area, that is what is from A here, A + dA is here and ds is along the streamlines and theta can consider is a rotations of with respect to the horizontal line, so we can find out dz will be the ds sin theta. Now let us have a basic concept here, where applying it that along a streamlines in frictionless flow. Again, I am to repeat to you. Whenever we talk about the Bernoulli equations, it has two assumptions with us.

That along a streamline and it is a frictionless flow, okay. So conservations of mass if I apply it through Reynolds transport theorem, okay. In this case, we have mass influx in and out is equal to change of the mass within this control volumes, that should be equal to zero. And that is what, as the density varies with the time, we can write in this form of this integral part, this integral part we can write in this form, okay? Here we consider density varies with respect to t.

$$\frac{d}{dt}\left(\int_{cv} \rho d\forall\right) + \dot{m}_{out} - \dot{m}_{in} = 0 \approx \frac{\partial \rho}{\partial t} d\forall + d\dot{m}$$

$$d\dot{m} = d(\rho A V) = -\frac{\partial \rho}{\partial t} A ds$$

$$\dot{m} = \rho A V$$

$$d\forall \approx A\, ds$$

It does not have a function with space much, so then we have this component and the rate of the challenge of the mass within the element of the control volumes, the mass flux will be rho times of AV, rho times of Q. That is what we have applied here. And since we consider A and V is equal to the A into ds, that is what is the volume part. These are very simple things, you just look it how to get it this part.

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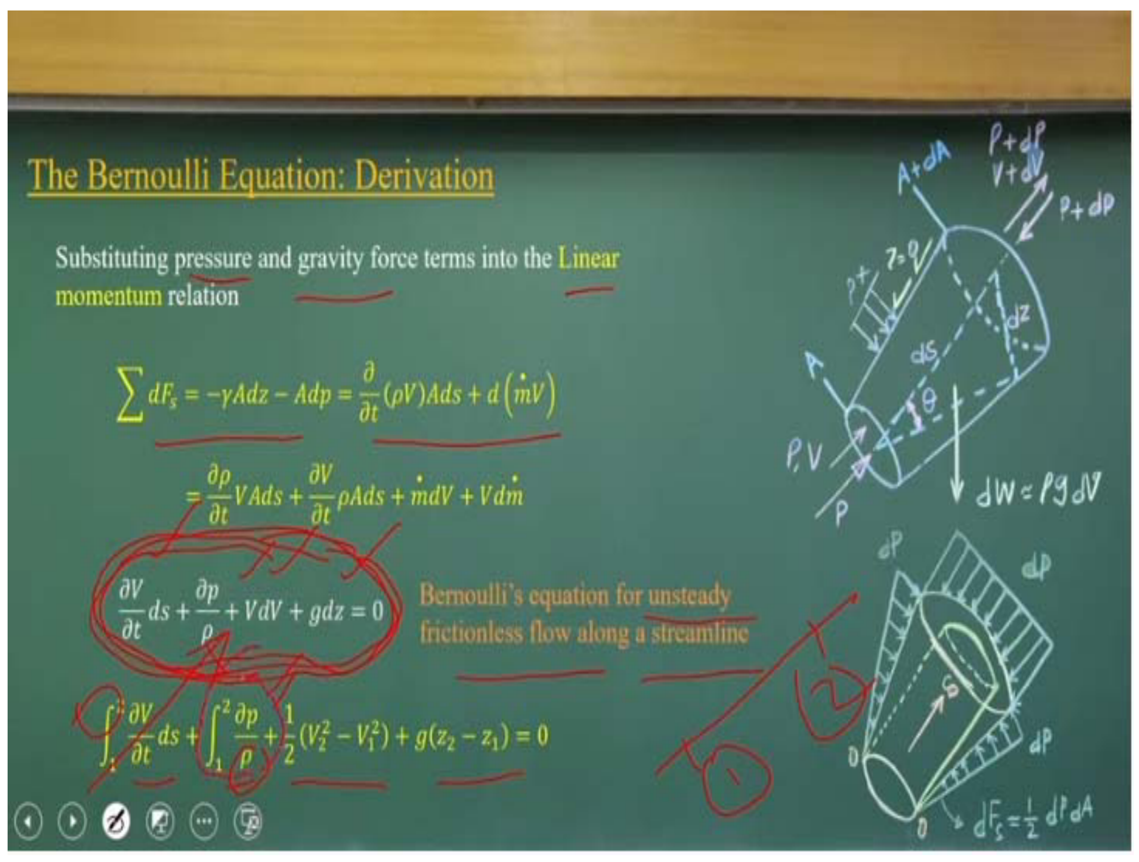

Now I am applying the linear momentum equations in streamline directions or streamwise directions. So sum of the force should be equal to the rate of change of the momentum in the control volume. Influx and the outflux, the momentum flux, the net what we will get at this part and this integrals we can simplify it into the part.

$$\sum dF_s = -\gamma Adz - Adp = \frac{\partial}{\partial t}(\rho V)Ads + d\left(\dot{m}V\right)$$

$$= \frac{\partial \rho}{\partial t}VAds + \frac{\partial V}{\partial t}\rho Ads + \dot{m}dV + Vd\dot{m}$$

Here is the dV, V cross is the volume, the elemental volumes will be A into ds and we have a ρ V. Both product is also, velocity is varies it in a space and the time, the densities so ρ V.

Then we have to find out the force due to the gravity.

$$dF_{s,garv} = -dW\sin\theta = -\gamma A\sin\theta = -\gamma Adz$$

That is what easily we can find out of $dW\sin\theta$ and this is the $\gamma A\sin\theta$ yet, that is what will give us the component of the gravity force is acting along the X directions. The gravity force component, wet component, the control volume along the X directions, we are equating along the streamline directions. Similar way, if I can equate the pressure in the slanted side of steamtube component, then we will also get it the net force.

$$\frac{\partial V}{\partial t}ds + \frac{\partial p}{\rho} + VdV + gdz = 0$$

With some approximations of neglecting smaller order terms, will get the Adp, so please look at the negative sign, negative signs are just looking the opposite of the directions what we have considered it. So, if I equate the force, the pressure force and gravity force component in linear momentum equations, will get it in this part, okay. Again will be just equating of these two part and if I just rearrange this part and look it, there are some of the terms are in mass conservation equations.

$$\int_1^2 \frac{\partial V}{\partial t}ds + \int_1^2 \frac{\partial p}{\rho} + \frac{1}{2}(V_2^2 - V_1^2) + g(z_2 - z_1) = 0$$

If I put it that part, I will get the Bernoulli equations in this form, which is for unsteady frictionless flow along a streamline. So, this is the equations. Along the streamlines, we will have the equations, which will be the pressure components will be there, the velocity component will be there, and the gravity force component will be there and momentum flux component will be there. So this component now will be simplified for steady flow. What we will do it, that we can integrated these things in a two points.

If I have two points, one and two locations, we can integrate over that, that is what we have done it here. So if I have two points, and we are looking the integrations of these two points, if you look at the integrations of these two points, we will come these, and this will be the integral of this part and this part.

$$dF_{s,press} = \frac{1}{2}dpdA - dp(A + dA) \approx -Adp$$

Now, this equation still has the integral for the pressure components, and also the local velocity components along the streamline directions. So we need to have again further assumptions to make it the standard form of Bernoulli's equations what you know.

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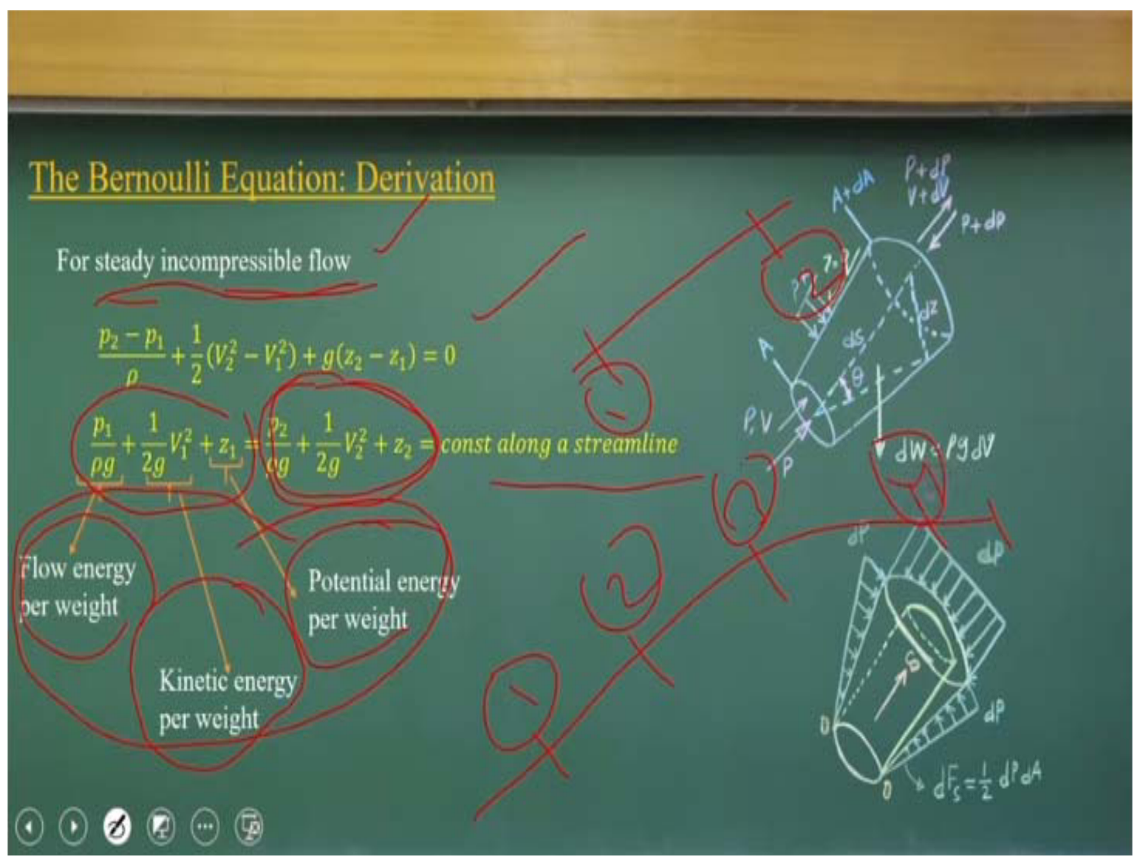

See what we can have, you have a steady incompressible flow, if I simplify that way. So as soon as you consider the steady incompressible flow, this component becomes zero. So any time derivative components becomes zero, parcel derivative components become zero. That is there, and when it is incompressible, that means the rho is a constant quantity. So it will go out from these integrations. Only we have a ρdV to integrate it. So these integrations is much easier now for the case where you have steady, incompressible flow.

$$\frac{p_2 - p_1}{\rho} + \frac{1}{2}(V_2^2 - V_1^2) + g(z_2 - z_1) = 0$$

Just rearrange this, I will have this form. So I am just rearranging the terms, which is a constant along the streamlines is one part, what is this? This is the flow energy per weight, I will explain it, what is that flow energy. This is the kinetic energy per weight, this is the potential energy per weight. So there is a three energy component at a section 1 that is what is called total energy.

It has a three component, one is a flow energy and one is due to the pressure, kinetic energy due to the velocities and the potential energy because of the positions, the z1. The height from the **(()) (34:03)** z1 height. Similar way, if you look at this spot, what is showing that? Again the total energy, one is a low energy because of the pressure, the kinetic energy because of the velocities field and you have a potential energy because of this.

So whenever you have a streamtube or the steamlines, any point you consider 1, 2, 3, 4. If it is a fictional loss, there is no energy losses in these 1, 2, 3, 4 points, we will have a total energy becomes constant and this total energy has three component, the flow energy because of the pressure, the kinetic energy because of the velocities. Like you have a, any solid mechanics, when you have in a motion you have a kinetic energy, it is the same things, but per weight. Potential energy per weight.

So, the sum of these energies, total energy per weight is remain same from 1 to 2, 2 to 3, 3 to 4, unless otherwise there is an energy loss between two locations of a streamline. So, that means, this is a very simplified equations for the fluid flow, fluid flow maybe the turbulent, maybe the laminar, but if we can identify the streamlines along a streamlines, if there is no energy is taking out or given to the systems, then energy becomes conserved, the total energy becomes conserved.

$$\frac{p_1}{\rho g} + \frac{1}{2g}V_1^2 + z_1 = \frac{p_2}{\rho g} + \frac{1}{2g}V_2^2 + z_2 = const\ along\ a\ streamline$$

That means in fluid flow problems, we have three energy components, flow energy that is because of the pressures, kinetic energy and the potential energy. The flow energy is a totally different as compared to the solid mechanics. Because in solid mechanics, we talk about kinetic energy and the potential energy, not the flow energy. But in case of the fluid flow or heat transfer, we talk about flow energy. I will discuss more about the flow energy when you talk virtual fluid balls concept.

Remember it, we introduced the virtual fluid valve concept. Now, I will talk that virtual fluid valve concept, why I am not talking the balls, I am talking of virtual fluid balls.

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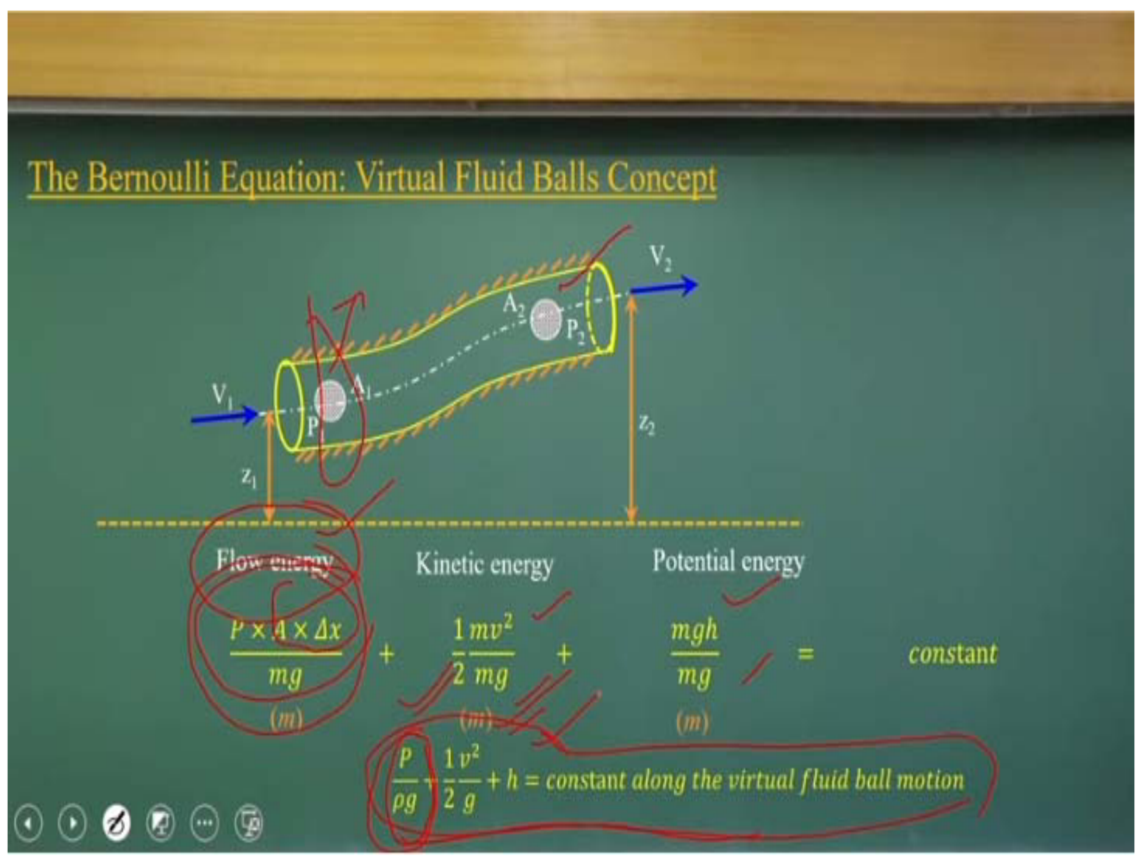
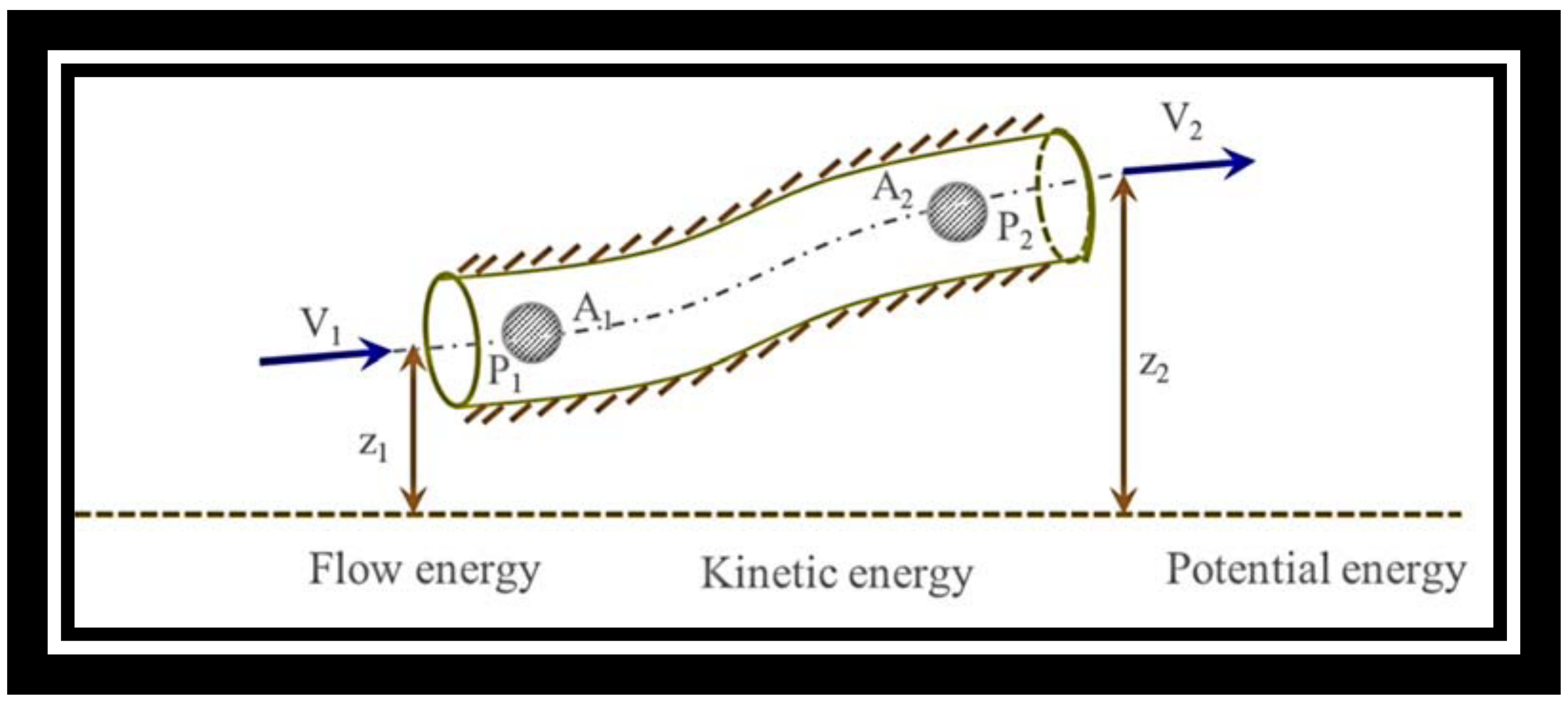


Because when I am talking about the ball at this locations, which are moving along the streamlines that p and the p2. Here the balls are moving it, it has potential energy, it has the kinetic energy, there is no doubt about that. But since it is a virtual fluid balls, that means here when you talk about any cross sections, there are a large number of balls are there. And as they are moving it, there are pressure field is in it over that. As the pressure field is working over that, what is the amount of energy is done because of this presser field.



$$\frac{P \times A \times \Delta x}{mg} \quad + \quad \frac{1}{2}\frac{mv^2}{mg} \quad + \quad \frac{mgh}{mg} \quad = \quad constant$$

That is what is called flow energy. So that component is there. That is the reasons we call virtual fluid balls, it is not the balls. It is the balls movement as a theoretically we are looking it, which has the flow energy because of the pressure field variations. Because of the number of the fluid balls are there, they are exerting a pressure on this particular ball. As this pressure is working it, what is the work? Force into the distance. That means pressure into the area of force into distance.

$$\frac{P}{\rho g} + \frac{1}{2}\frac{v^2}{g} + h = constant\ along\ the\ virtual\ fluid\ ball\ motion$$

That is what the volume of the translations. P into V by wet mg that is what is the p rho g. If you look it this part, the conceptually you will try to understand it because of the pressures, and if I have area of the ball of A, this is doing a work done, force into distance is theoretical delta x, the amount of energy to do this work will be the flow energy. That is what is p rho gs. The kinetic energy per weight. The half mV square by mg. That is will be come it, you need to look the meters, again you look it.